10.3. Some Partial Differential Equations

1181. The Laplace Equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

applies to potential energy function $u(x,y)$ for a conservative force field in the xy-plane. Partial differential equations of this type are called elliptic.

1182. The Heat Equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial u}{\partial t}$$

applies to the temperature distribution $u(x,y)$ in the xy-plane when heat is allowed to flow from warm areas to cool ones. The equations of this type are called parabolic.

1183. The Wave Equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial t^2}$$

applies to the displacement $u(x,y)$ of vibrating membranes and other wave functions. The equations of this type are called hyperbolic.